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The electromagnetic field with a vertical magnetic dipole in a three-layered

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ABSTRACT: The effect of the sea bottom cannot be neglects in some situations, for example in the case of a low frequency, a shallow sea, and the access of transmitter or receiver in the sea bottom. The effect of the sea bottom on radio wave propagation is ascertained theoretically. And it will act as an important part in such cases. Two integrals transformation of the wave equations of Hertizan vector-a two dimensional Fourier transform in the horizontal coordinates in space are applied. The integral representation determines the electromagnetic field anywhere. Considering a three layered sea, the multipath reflection and multilateral wave that cannot be generated in a two-layered media are obtained.

***Keywords*:** Electromagnetic field; Magnetic dipole.

INTRODUCTION

Communication in the sea has been required for the human activities that take place there for many years, many writers. Wait [1], Moore and Blair [2] and Durrani [3] have considered this problem, however, in their studies, the sea was assumed to be semi-infinite. This assumption is not a good approximation in the case of shallow sea, low frequency wave and the presence of transmitter near the bottom of the sea in their cases. In several recent publications, however, the case is considered where the time dependence of the current in the dipole is impulsive rather than harmonic e.g. the studies by Lindell and Alanen[4] and Dvorak and Mechaik[5], these techniques can be grouped in the following categories; quasi-analytical solutions which include asymptotic approximations series expansions and image representations, direct numerical integration, and methods which use numerical techniques. Abo Seliem[6], theoretical study for computing the magnetic field from a Fitzgrald vector in the ionosphere is presented Zedan and Abo Seliem [7] Also, Abo Seliem[8] which has been simplified by De Hoop and Frankena[9] studied the electromagnetic field due to a dipole placed within a uniform Saddle point and residue methods are used to compute the integral. The problem is evaluated mathematically using the residue and Saddle point methods and integrals along branch cuts show lateral waves.

FUNDAMENTALl EQUATIONS

In Fig.1, a small loop antenna, whose magnetic moments is ISo is located in the middle layer at depth d_1, we suppose that the thickness of the sea is a, and that air is infinite upward and the ground downward along the x-axis. The magnetic permeability is taken equal to that of the free space is every layer. γ_2 and γ_3 can be written as follows:

$$\gamma_i \cong \sqrt{\mu_o \sigma_i \omega / 2}(1-j) \equiv \gamma_i^{'}(1-j), \quad i = 2,3 \qquad (1)$$

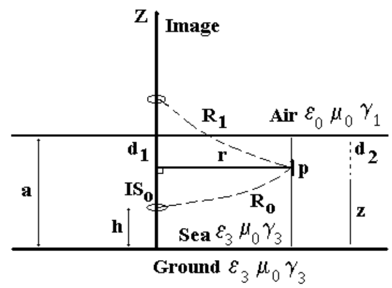

Figure 1. Geometric configuration of position

The magnetic Hertz vector has only a Z-component π_z^*. A particular solution of the non homogeneous wave equation is obtained first by the infinite space Green's function, and it is expressed in integral from Stranton [10]. The Hertz vector in each region can be obtained by using a Sommerfeld integral as follows, considering that region I and III are infinite upward and down ward, respectively, along the Z-axis.

$$\pi_{Z1} = \frac{ISo}{4\pi}\int_0^\infty \Phi_1(\lambda)\exp(-\gamma_1 z)J_0(\lambda r)d\lambda \qquad (2)$$

$$\pi_{Z2} = \frac{ISo}{4\pi}\int_0^\infty [\frac{\lambda}{\gamma_2}\exp(-\gamma_2|z-h|) + \Phi_2(\lambda)\exp(\gamma_2 z) + \Phi_3(\lambda)\exp(-\gamma_2 z)]J_0(\lambda r)d\lambda \qquad (3)$$

$$\pi_{Z3} = \frac{ISo}{4\pi}\int_0^\infty \Phi_4(\lambda)\exp(\gamma_3 z)J_0(\lambda r)d\lambda \qquad (4)$$

Where

$\gamma_i = \sqrt{\lambda^2 - k_i^2}$, $(i = 1,2,3)$, $J_0(\lambda r)$ is the Bessel function of order zero.

The tangent components of the electric and magnetic field are continuos at the Boundary,

$$\pi_{Z2} = \pi_{Z3} \quad \text{and} \quad \frac{\partial \pi_{Z2}}{\partial z} = \frac{\partial \pi_{Z3}}{\partial z} \quad \text{at} \quad z = 0 \qquad (5)$$

$$\pi_{Z1} = \pi_{Z2} \quad \text{and} \quad \frac{\partial \pi_{Z1}}{\partial z} = \frac{\partial \pi_{Z2}}{\partial z} \quad \text{at} \quad z = a$$

We shall deal with the field in region II (sea). We find then by combining the boundary condition (5) with (2-4), and then they are as follows:

$$\Phi_2(\lambda) = \frac{\lambda[(\gamma_2 - \gamma_3)\exp(-\gamma_2(h+a)) + (\gamma_2 + \gamma_3)\exp(\gamma_2(h-a))]}{\gamma_2[C_{12}(\gamma_2 + \gamma_3)\exp(\gamma_2 a) + (\gamma_2 - \gamma_3)\exp(-\gamma_2 a)]} \qquad (6)$$

$$\Phi_3(\lambda) = \frac{\lambda[C_{12}(\gamma_2 - \gamma_3)\exp(\gamma_2(a-h)) - (\gamma_2 - \gamma_3)\exp(\gamma_2(h-a))]}{\gamma_2[C_{12}(\gamma_2 + \gamma_3)\exp(\gamma_2 a) + (\gamma_2 - \gamma_3)\exp(-\gamma_2 a)]} \qquad (7)$$

where $C_{12} = \frac{(\gamma_1 + \gamma_2)}{(\gamma_1 - \gamma_2)}$. We transform $J_0(\lambda r)$ into Hankel function $H_0^{(2)}(\lambda r)$ to change the semi-infinite integral in (3) into a fully infinite integral:

$$\pi_{Z2} = \frac{ISo}{8\pi}\int_{-\infty}^\infty [\frac{\lambda}{\gamma_2}\exp(-\gamma_2|z-h|) + \Phi_2(\lambda)\exp(\gamma_2 z) + \Phi_3(\lambda)\exp(-\gamma_2 z)]H_0^{(2)}(\lambda r)d\lambda \qquad (8)$$

The electric and magnetic fields are derived from the Hertz vector as is well known in the present circumstances $\mathbf{E}$ and $\mathbf{H}$ in the sea corresponding to π_{Z2} are as follows:

$$H_r = \frac{-ISo}{8\pi}\int_{-\infty}^\infty [\frac{\lambda}{\gamma_2}\exp(-\gamma_2|z-h|) + \Phi_2(\lambda)\exp(\gamma_2 z) + \Phi_3(\lambda)\exp(-\gamma_2 z)]\gamma_2\lambda H_0^{(2)}(\lambda r)d\lambda \qquad (9)$$

$$E_\Phi = \frac{-j\omega\mu_0 ISo}{8\pi}\int_{-\infty}^\infty [\frac{\lambda}{\gamma_2}\exp(-\gamma_2|z-h|) + \Phi_2(\lambda)\exp(\gamma_2 z) + \Phi_3(\lambda)\exp(-\gamma_2 z)]\lambda H_0^{(2)}(\lambda r)d\lambda \qquad (10)$$

$$H_z = \frac{ISo}{8\pi}\int_{-\infty}^\infty [\frac{\lambda}{\gamma_2}\exp(-\gamma_2|z-h|) + \Phi_2(\lambda)\exp(\gamma_2 z) + \Phi_3(\lambda)\exp(-\gamma_2 z)]\lambda^2 H_0^{(2)}(\lambda r)d\lambda \qquad (11)$$

And

$$E_r = E_z = H_\Phi = 0 \qquad (12)$$

In this paper we will show the process of estimating E_Φ, H_r and H_z are also estimated in a similar way paying attention to (9), we are sure that the first term of the integral is the direct wave from the source of the observing

point. This term is important if the transmitter and receiver are set very closely. If the two points are separated for from each other, this term will vanish because the propagation path is in the sea and the attenuation is very large. The last term is regarded as similar to the second term except for $\exp(-\gamma_2 z)$.
We treat the second term, because we suppose that the observing point is near the surface of the sea. Here we focus our attenuation on the second term, and we write it again as:

$$H_r^{(1)} = \frac{-ISo}{8\pi}\int_{-\infty}^{\infty}\frac{\lambda^2}{\gamma_2}\frac{(\gamma_1-\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 h)}{(\gamma_1+\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 a)} \exp(-\gamma_2(d_1+d_2))H_0^{(2)}(\lambda r)d\lambda \qquad (13)$$

In, $\gamma_i\ (i=1,2,3)$ is not single-valued.

RESULTS AND DISCUSSION

Discussion of the equation

The evaluation of the integral (13) is a very deficient task. Therefore, using Bessel integral representation:then

$$H_r^{(1)} = \frac{-ISo}{8\pi}\int_{-\infty}^{\infty}\frac{M(\lambda)}{N(\lambda)}H_1^{(2)}(\lambda r)\lambda^2 d\lambda \qquad (14)$$

Where

$$M(\lambda) = [(\gamma_1-\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 h)]\exp(-\gamma_2(d_1+d_2)) \qquad (15)$$

And

$$N(\lambda) = \gamma_2[(\gamma_1+\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 a)] \qquad (16)$$

The integral in (14) is not single-valued because of the square roots of $\alpha_i\ (i=1,2)$ that upper in it. Corresponding to four combinations of signs of α_i, the integral has four values and its Riemann surface has four sheets.

To insure the convergence of our integrals, we demand that the path of integration, at infinity, should be on the permissible sheet only. Besides, the branch points $\lambda = \pm k_1$ and $\pm k_2$, then is a further singularity of the integrand in (14), these poles lies in the second and fourth quarters of the complex $\lambda-$plane. Their positions are between the branch cuts at $\pm k_2$, and an imaginary axis (Fig.2.)

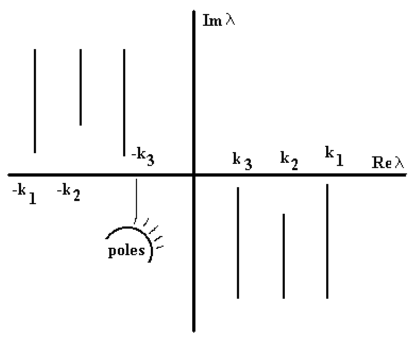

Figure 2. Branch cuts and poles in the complex λ-plane

The integral (14) cannot be solved exactly, so we apply the residue and Saddle point methods, we can put it in the form

$$H_r^{(1)} = \frac{-ISo}{8\pi}\int_{-\infty}^{\infty}\frac{M(\lambda)}{N(\lambda)}H_1^{(2)}(\lambda r)\lambda^2 d\lambda \qquad (17)$$

The previous integral (17) can be evaluated along the contour, from $-\infty$ to ∞, and its values go around the poles and branch cuts. An equation (11) then takes the form:

$$H_r^{(1)} = \frac{-ISo}{8\pi}\sum\left(\frac{M(\lambda)}{N(\lambda)}H_1^{(2)}(\lambda r),\lambda_s\right) - \frac{ISo}{2\pi}\int_0^{\infty}\frac{M(\lambda)}{N(\lambda)}H_1^{(2)}(\lambda r)\lambda^2 d\lambda \qquad (18)$$

Where $n(\alpha)$'s are eigenvalues of the poles of the integrand and λ_s is the solution of the poles equation

$$N(\lambda) = \gamma_2[(\gamma_1+\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 a)] = 0 \qquad (19)$$

This determines the poles of integral, substituting the value $\lambda = \lambda_s$ in the first term (18), where

$$\mathbf{D}^{k}(\lambda)=\frac{-\mathbf{ISo}}{\pi}\left[\frac{\mathbf{M}(\lambda)}{\mathbf{N}(\lambda)}\mathbf{H}_1^{(2)}(\lambda)\right],\quad \mathbf{N}'(\lambda)=\left[\frac{\partial \mathbf{N}(\lambda)}{\partial\lambda}\right]_{\lambda=\lambda_s} \qquad (20)$$

Next, we can estimate the second term in (12) by using the Saddle point method. In this work, we treat the far field so that the Hankel function can be transformed into asymptotic expansion, as well-know [10].

$$\mathbf{H}_1^2(\lambda)=\sqrt{2/(\pi\lambda r)}\ \exp(-j(\lambda r-3\frac{\pi}{4})) \qquad (21)$$

From (16) and (17), we can get the following:

$$\mathbf{H}=\int\sqrt{2/(\pi\lambda r)}\ \exp(-j(\lambda r-3\frac{\pi}{4}))\sqrt{\frac{\lambda}{\gamma_2}}\,A(\lambda)\exp(-\gamma_2(2h-d_1-d_2))d\lambda \qquad (22)$$

Where

$$A(\lambda)=\frac{(\gamma_1-\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 h)}{(\gamma_1+\gamma_2)(\gamma_3+\gamma_2)+(\gamma_1-\gamma_2)(\gamma_2-\gamma_3)\exp(-2\gamma_2 a)} \qquad (23)$$

We set $\mathbf{r}=\mathbf{R}\sin\theta$ and $z-(d_1+d_2)=\mathbf{R}\cos\theta$, then $\mathbf{H}$ take the form:

$$\mathbf{H}=\int\sqrt{2/(\pi R\sin\theta)}\ \exp(-jRg(\lambda))\Phi(\lambda)\exp(-3j\pi/4)\,d\lambda \qquad (24)$$

This, the Saddle point $\lambda=\lambda_s$ for the integral is determined by [11]

$$g'(\lambda)=\sin\theta+\frac{j\lambda\cos\theta}{\sqrt{\lambda^2-k_2^2}} \qquad (25)$$

Therefor $\lambda_s=k_2\sin\theta$ and $\Phi(\lambda)=A(\lambda)\exp(jk_2\cos\theta\sqrt{k_2\sin\theta})$

Then, the integral (22) can be evaluated as follows:

$$\mathbf{H}=\frac{A(\lambda_s)\exp(jk_2 R)}{R} \qquad (26)$$

Where $R=\sqrt{r^2+(z-d_1-d_2)^2}$, $\theta=\sin^{-1}\frac{r}{R}$.

Discussion of Plot

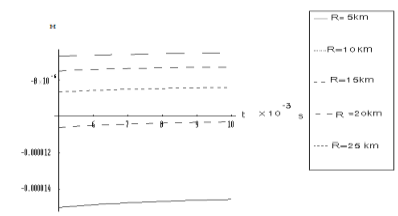

Figure 3. The relationship between the value of magnetic field strength and the time.

Figure (3): describes the relationship between value of the magnetic field strength and the time .In this figure (saturation relationship) where value of imaginary is constant at varying time for each value of R but we note that at r = 5 km and 10 km the saturation curves are negative (-ve) value . But for R = 15 km, 20 km and 25 km, d1=d2=3 km. the saturation curve are positive (+ ve) value. he integrals of the electromagnetic field are solved and the results are represented graphically by using the description of the polarization dependence of the time .Physically the integral representation in zero denominator of these integral, of the secondary field for medium (2) (sea), apply a similar way treatment to the denominator of the integral representation for the medium (2) in case uniform duct, we sort out the singularities of these integral on knowing , poles , branch points and which will help in determining the branch cut are discusse.

CONCULSION

The problem of communication in the sea has been considered by many writers, the propagation of radio waves in the sea is of great importance in many practical applications. Considerable effort and speculation have thus been devoted to establish the theoretical fundamental for such problem.

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